My notes on steady state changes in CBF and CBV

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This note is based on the paper by

Lee, S-P, Duong, T. Q., Yang, G., Iadecola, C., Kim, S-G. (2001) Relative changes of cerebral arterial and venous blood volumes during increased cerebral blood flow: implications for BOLD fMRI. Magnetic Resonance in Medicine, 45, 791-800.

The paper by Lee (2001) investigated the relative changes of cerebral arterial and venous blood volumes with respect to increases in cerebral blood flow. In Figure 4 of the paper, they estimated the relationship between the total blood volume and flow changes as

$$\frac{V_T}{V_{T0}} = 0.31 * \frac{F}{F_0} + 0.68 \tag{1}$$

where the baseline is at

$$F_0 = 58\, ml/100g/\min$$

Furthermore in Figure 6, they estimated the relationship between the arterial blood volume fraction and the blood flow as

$$\frac{V_a}{V_T} = 0.002F + 0.137 \tag{2}$$

for CBF in the range (40, 100) ml/100g/min. Note that the volume fraction between the artery and the total CBV changes as CBF changes, but the slope is relatively small. At the baseline $F_0 = 58\, ml/100g/\min$, the volume ratio $V_{a0}/V_{T0} = 0.253$. As CBF varies between 40~100, this volume ratio varies between 0.2~0.4.

From Figures 4 and 6, Lee et al plotted Figure 7 (a, b) and estimated various relationships between volume and flow in the arterial and venous compartments. We show here that these relationships can also be estimated based on the above equations alone.

First of all, Figure 7a of Lee et al showed the relationship between V_a/V_{T0}, and V_v/V_{T0} as a function of F/F_0. From eqn.s (1) and (2) above, we have:

$$
\begin{aligned}
\frac{V_a}{V_{T0}} &= \frac{V_a}{V_T}\frac{V_T}{V_{T0}} \\
&= (0.002F + 0.137) * \left(0.31\frac{F}{F_0} + 0.68\right) \\
&= \left(0.002\frac{F}{F_0}F_0 + 0.137\right) * \left(0.31\frac{F}{F_0} + 0.68\right)
\end{aligned}
\tag{3}
$$

At baseline $F_0 = 58\,ml/100g/\min$, the above equation becomes

$$
\begin{aligned}
\frac{V_a}{V_{T0}} &= \left(0.116\frac{F}{F_0} + 0.137\right) * \left(0.31\frac{F}{F_0} + 0.68\right) \\
&= 0.036\left(\frac{F}{F_0}\right)^2 + 0.1214\frac{F}{F_0} + 0.0932
\end{aligned}
\tag{6}
$$

This is in fact a quadratic equation. However within the range $F/F_0 \in [0.8, 1.6]$, the quadratic effect is almost negligible compared with the linear effect, as shown in my Figure 1 below (circles). In particular, the changes in CBF is related to the changes in arterial volume fraction as

$$
\begin{aligned}
\left.\frac{\Delta V_a}{V_{T0}}\right|_{F=F_0} &= 0.036 * 2 * \left.\frac{F}{F_0}\right|_{F=F_0}\frac{\Delta F}{F_0} + 0.1214\frac{\Delta F}{F_0} \\
&= (0.072 + 0.1214)\frac{\Delta F}{F_0} \\
&= 0.1934\frac{\Delta F}{F_0}
\end{aligned}
\tag{7}
$$

This matches well to the linear fit: $V_a/V_{T0} = 0.20 * F/F_0 + 0.05$ given by Lee et al (2001) in Figure 7a, which will yield $\Delta V_a/V_{T0} = 0.20 * \Delta F/F_0$.

In Lee's paper, it was assumed that capillary volume is zero. Based on this assumption, Lee estimated all venous volume fractions. Here we will first use Lee's assumption to verify the results in Figure 7 of Lee's paper. Then we will discuss the alternative assumption when the non-zero capillary volume is taken into account.

Assuming zero capillary volume, the venous volume is given by $V_v = V_T - V_a$. Hence

$$\begin{aligned}\frac{V_v}{V_{T0}} &= \left(\frac{V_T - V_a}{V_T}\right)\frac{V_T}{V_{T0}} = \left(1 - \frac{V_a}{V_T}\right)\frac{V_T}{V_{T0}} \\ &= \frac{V_T}{V_{T0}} - \frac{V_a}{V_{T0}} \\ &= \left(0.31\frac{F}{F_0} + 0.68\right) - \left(0.036\left(\frac{F}{F_0}\right)^2 + 0.1214\frac{F}{F_0} + 0.0932\right) \\ &= -0.036\left(\frac{F}{F_0}\right)^2 + 0.1886\frac{F}{F_0} + 0.5868\end{aligned} \tag{8}$$

Again the relationship is quadratic, but as seen in my Figure 1 (stars), within the CBF range of interest, the quadratic effect is almost negligible. At baseline $F_0 = 58\,ml/100\,g/\min$,

$$\left.\frac{\Delta V_v}{V_{T0}}\right|_{F=F_0} = 0.1166\frac{\Delta F}{F_0} \tag{9}$$

Compared with the linear relationship: $V_v/V_{T0} = 0.11 * F/F_0 + 0.63$ given by Lee et al, we see that this linear equation will result in $\Delta V_v/V_{T0} = 0.11 * \Delta F/F_0$ which matches well with the above equation (9).

In Figure 7b, Lee et al. tried to relate CBF changes to the volume fractional changes normalised not with respect to the total blood volume, but with respect to the blood volume in the relevant compartment. These relationships can be obtained as follows.

$$\frac{V_a}{V_{a0}} = \frac{V_a}{V_{T0}}\frac{V_{T0}}{V_{a0}} = \frac{0.036(F/F_0)^2 + 0.1214(F/F_0) + 0.0932}{0.002F_0 + 0.137} \tag{10}$$

At baseline $F_0 = 58\,ml/100\,g/\min$, this becomes

$$\frac{V_a}{V_{a0}} = \frac{0.036(F/F_0)^2 + 0.1214(F/F_0) + 0.0932}{0.253} \tag{11}$$

and the relationship between changes in normalised arterial CBV and normalised CBF is

$$\frac{\Delta V_a}{V_{a0}} = 0.7644\frac{\Delta F}{F_0} \tag{12}$$

Lee et al estimated from Figure 7b that $V_a/V_{a0} = 0.79(F/F_0) + 0.19$. This will give $\Delta V_a/V_{a0} = 0.79 * \Delta F/F_0$ which is comparable with eqn.(12).

The ratio V_v/V_{v0} can be easily related to F/F_0 if one assumes zero capillary volume, as we note that

$$\frac{V_v}{V_{v0}} = \frac{V_v}{V_{T0}} \frac{V_{T0}}{V_{v0}} \tag{13}$$

$\frac{V_v}{V_{T0}}$ is given by eqn.(8), and $\frac{V_{v0}}{V_{T0}} = \frac{V_{T0} - V_{a0}}{V_{T0}} = 1 - \frac{V_{a0}}{V_{T0}}$. Again at baseline $F_0 = 58\, ml/100g/\min$, $\frac{V_{v0}}{V_{T0}} = 1 - 0.253 = 0.747$. After simple manipulation, it can be shown that

$$\frac{\Delta V_v}{V_{v0}} = 0.156 \frac{\Delta F}{F_0} \tag{14}$$

Again this is comparable with the relationship $\frac{\Delta V_v}{V_{v0}} = 0.15 \frac{\Delta F}{F_0}$ given by Lee et al.

Equations (1) and (12) enable us to relate the arterial fractional CBV changes to the total fractional CBV changes as

$$\frac{\Delta V_a}{V_{a0}} = \frac{0.7644}{0.31} \frac{\Delta V_T}{V_{T0}} = 2.466 \frac{\Delta V_T}{V_{T0}} \tag{15}$$

If one makes the assumption made by Lee et al that there is zero capillary blood volume, then the venous fractional CBV changes to the total fractional CBV changes is given by equations (1) and (14) as

$$\frac{\Delta V_v}{V_{v0}} = \frac{0.156}{0.31} \frac{\Delta V_T}{V_{T0}} = 0.503 \frac{\Delta V_T}{V_{T0}} \tag{16}$$

However we know that the capillary volume is non-zero. Thus if we take this into account but still assume that the *change* in CBV of the capillary is zero, then

$$\frac{\Delta V_v}{V_{v0}} = \frac{\Delta V_T - \Delta V_a}{\Delta V_{T0}} \frac{\Delta V_{T0}}{V_{v0}} = \frac{\Delta V_T}{V_{T0}} \frac{1 - r_a p_{a0}}{1 - p_{a0} - p_{c0}} \tag{17}$$

where $r_a = \dfrac{\Delta V_a / V_{a0}}{\Delta V_T / V_T}$, $p_{a0} = \dfrac{V_{a0}}{V_{T0}}$, and $p_{c0} = \dfrac{V_{c0}}{V_{T0}}$. From eqn.(2) we know that at baseline $F_0 = 58\,ml/100g/\text{min}$, $p_{a0} = 0.253$. Furthermore eqn.(15) gives $r_a = 2.466$. If we assume that $p_{c0} = 0.25$, then

$$\frac{\Delta V_v}{V_{v0}} = \frac{\Delta V_T - \Delta V_a}{\Delta V_{T0}} \frac{\Delta V_{T0}}{V_{v0}} = 0.757 \frac{\Delta V_T}{V_{T0}} \tag{18}$$

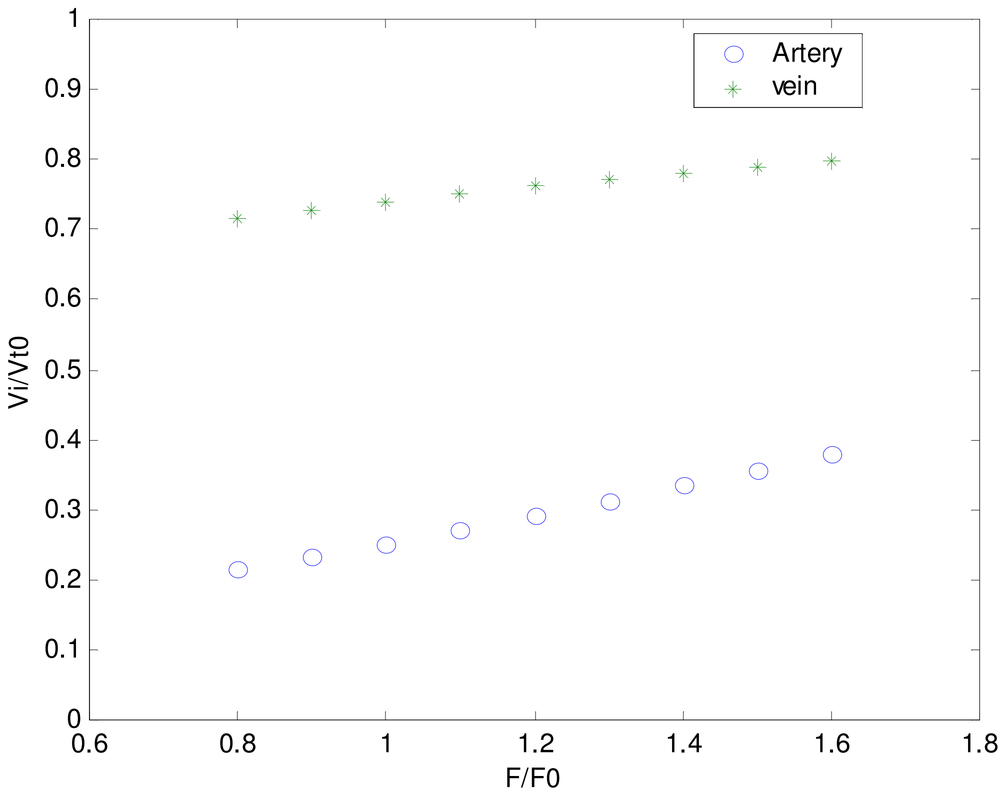

Figure 1