COMPUTERS IN UNDERGRADUATE MATHEMATICS TEACHING

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1. Philosophy

A significant proportion of the questions that we now teach our students to answer in exams or on problem sheets, can now be answered by a computer system such as Maple. It seems clear that this fact should make some kind of difference to the way that we teach mathematics, and the selection of topics that we teach. The details are much less clear, but I will make an attempt to sketch them out.

My suggestion is that in the medium term, we should teach our students to use Maple fluently, both as a tool to solve mathematical problems, and as an aid to understanding traditional mathematical theory. These aims should be deeply embedded in the degree programme, with a significant amount of Maple work in every course whose material permits it. In the rest of these notes, I sketch how a degree programme based on these ideas might look, and give arguments in support of it.

2. Aims

What are our aims? How should we decide what to teach?

(a) We should teach mathematics that we consider to be interesting and challenging. We should try to convey some of the beauty of mathematics, and perhaps its deep cultural and historical roots. We are more likely to succeed in this if we teach subjects with some relation to our research.
(b) We should try to instill the intellectual skills and habits characteristic of mathematics:
  (i) finding patterns, and precise statements to encapsulate them
  (ii) generalisation and axiomatisation
  (iii) experimentation with typical cases, special cases, and extreme cases; finding examples and counterexamples
  (iv) finding theorems to apply, choosing data to apply them to, verifying conditions, extracting conclusions
  (v) reading and understanding proofs; criticising incorrect proofs; constructing proofs
  (vi) understanding and using algorithms.
(c) Where consistent with (a), we should teach mathematics that may be directly useful to students who go on to employment in finance, science, engineering, computing, or other areas that might require mathematics.
(d) We should teach transferrable skills that may be useful more generally: teamwork, communication and presentation, programming, other ICT skills.

3. Algorithms

Maple can carry out many essentially algorithmic processes that we teach the students to carry out: differentiation and integration, solution of systems of equations, evaluation of limits, and so on. Naively, one could conclude that we no longer need to teach these processes. I think it is important to understand precisely what is wrong with this conclusion, and to reconsider our approach and choice of topics in the light of that understanding.

(a) The methods used for these problems are interesting mathematics in their own right.

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(b) Knowing how to integrate (for example) helps students to understand what integration means.
(c) The software often gives answers that are misleading unless analysed intelligently. Occasionally, it gives answers that are definitively incorrect.
(d) Mathematics graduates ought to understand what they are doing, not just blindly trust a machine.
(e) Algorithms are something that weak but diligent students can learn successfully; removing them could lead to more failures.

Some possible conclusions:

(a) Problems to be done by hand should be chosen for their instructive nature. There should be fewer matrices with random entries, polynomials with random coefficients and so on; instead, we should use examples with interesting symmetry, physical meaning, or other mathematical significance. Alternatively, we could set problems with random matrices, have the computer do the grungy work of row-reducing them, and leave the students the task of drawing intelligent conclusions.
(b) Where we might previously have asked students to do one example by hand, we might now ask them to do half a dozen by computer, and to look for patterns in the answers; this may well give more insight and understanding.
(c) Where we might previously have asked students to do a problem algebraically, we might now ask them to check their answer by inserting random numbers, or by producing some kind of picture. Again, well-designed exercises of this kind have great potential to enhance understanding.
(d) Often we have problems that depend on a parameter n; only the first few cases are small enough to do by hand; but only when n is large does a clear pattern emerge. Here a very natural approach is to do one or two cases by hand, and then some more by computer.

4. Mathematical employment

I have no good data about the extent to which our graduates use mathematics in their careers. I conjecture that:

(a) A reasonable number of them work regularly with numerical data (often financial figures) which they need to process and analyse in fairly straightforward ways, and display graphically. They will probably use spreadsheets for this. There is a small amount of scope for using Maple to do more powerful processing, to automate some procedures and so on, but it would be unrealistic to overemphasise this. On the other hand, experience with Maple would probably make spreadsheets seem very easy and natural. I don't know where most people learn to use spreadsheets.
(b) A fair number of them work as programmers or software engineers. A mathematics degree probably develops analytical skills that are useful in this context. This effect would probably be enhanced if the use of Maple (as a programming system as well as a calculator) were integrated into the course.
(c) A few of them may work in computer games or animation. I have heard of some heavy-duty applications of Maple in these areas. Similarly, there are some other software niches with important mathematical content, such as computational geometry for computer aided design and manufacture.
(d) A few of them work as engineers, scientists, economists, or financial mathematicians. I anticipate that these areas will make rapidly increasing use of systems such as Maple. For professionals who lack time, lack practise, and need accurate answers, it is simply no longer rational to do intricate calculations by hand. Although Maple does numerical calculations more slowly than dedicated programs written in FORTRAN (for example), it is often much more efficient when one takes the programming time into account.

5. Transferrable skills

It is debatable whether teaching with Maple would impart any transferrable skills, beyond the rather direct ones discussed above. General familiarity with computers and programming may count as a transferrable skill.

6. Practicalities

How would we run a degree programme with tightly integrated use of Maple?

(a) Many staff would need training and practise with Maple.
(b) Many lectures would need to be held in rooms with video projectors, and the lecturers would need laptops. I would envisage a mixture of "chalk and talk" with occasional demonstrations of Maple calculations. Given that the laptop and projector would be in place anyway, some lecturers might prefer to use PowerPoint slides instead of a blackboard.
(c) Students would need initial training in the use of Maple, probably with quite intense tuition. I believe that some American universities make a lot of use of senior undergraduates as tutors in this context; I don't know if that is an option for us.
(d) Many courses would include homework, either regularly or occasionally, that involves the use of Maple. Students would be free to do this homework whenever they want. There would be various timetable slots in which a tutor would be available in a room full of PC's, ready to answer questions about all Maple assignments that had been set in the previous week. For at least a proportion of the assignments, students would be encouraged to help each other.
(e) Some technical work might be necessary to allow students to submit work or ask questions by email in a convenient way.

7. Other uses of computers

I anticipate that

(a) We will increasingly distribute lecture notes, problem sets and so on on the web.
(b) Some lecturers will also place Java applets, coloured or animated diagrams, and other nontraditional material on the web. They will also show such material in lectures using a video projector. (Although not closely related to the issues immediately under consideration, I think that the department should actively encourage these developments, and bid for resources from the Learning Media Unit to support them.)
(c) We may suggest or require that students write their projects using LaTeX. It may or may not be reasonable to allow Maple worksheets as an alternative.
(d) We will use computer aided assessment in various courses.

We will need to think about the interaction between all these things and our use of Maple.