```
> p := 2;
  v[0] := p;
  N := 3;
```

$$p := 2$$
$$v_0 := 2$$
$$N := 3 \quad \textbf{(1)}$$

Araki log coefficients

```
> LA := proc(n)
   option remember;
   if (n = 0) then
    return(1);
   else
    return(expand(add(LA(k)*v[n-k]^(p^k),k=0..n-1)/(p-p^(p^n))));
   fi;
  end:
```

Hazewinkel log coefficients

```
> LH := proc(n)
   option remember;
   if (n = 0) then
    return(1);
   else
    return(expand(add(LH(k)*v[n-k]^(p^k),k=0..n-1)/p));
   fi;
  end:
```

$F(x)$ will be $\exp_H(\log_A(x))$

```
> F := expand(add(C[i]*x^i,i=1..p^N));
```

$$F := C_1 x + C_2 x^2 + C_3 x^3 + C_4 x^4 + C_5 x^5 + C_6 x^6 + C_7 x^7 + C_8 x^8 \quad \textbf{(2)}$$

$FP[k] = F(x)^k$

```
> FP[0] := 1;
  for i from 1 to p^N do
   FP[i] := rem(expand(F * FP[i-1]),x^(p^N+1),x);
  od;
```

$$FP_0 := 1$$

$$FP_1 := C_1 x + C_2 x^2 + C_3 x^3 + C_4 x^4 + C_5 x^5 + C_6 x^6 + C_7 x^7 + C_8 x^8$$

$$FP_2 := \left(C_4^2 + 2\,C_3\,C_5 + 2\,C_1\,C_7 + 2\,C_2\,C_6\right) x^8 + \left(2\,C_3\,C_4 + 2\,C_2\,C_5 + 2\,C_1\,C_6\right) x^7 + \left(2\,C_2\,C_4\right.$$

$$+2\,C_1\,C_5+C_3^2\big)\,x^6+\big(2\,C_1\,C_4+2\,C_2\,C_3\big)\,x^5+\big(2\,C_1\,C_3+C_2^2\big)\,x^4+2\,C_1\,x^3\,C_2+C_1^2\,x^2$$

$$FP_3 := \big(3\,C_1^2\,C_6+3\,C_2\,C_3^2+6\,C_4\,C_1\,C_3+6\,C_1\,C_2\,C_5+3\,C_4\,C_2^2\big)\,x^8+\big(3\,C_3\,C_2^2+3\,C_3^2\,C_1+3\,C_1^2\,C_5+6\,C_4\,C_1\,C_2\big)\,x^7+\big(3\,C_4\,C_1^2+C_2^3+6\,C_2\,C_1\,C_3\big)\,x^6+\big(3\,C_1^2\,C_3+3\,C_1\,C_2^2\big)\,x^5+3\,C_1^2\,x^4\,C_2+C_1^3\,x^3$$

$$FP_4 := \big(12\,C_1\,C_3\,C_2^2+12\,C_4\,C_1^2\,C_2+C_2^4+4\,C_1^3\,C_5+6\,C_1^2\,C_3^2\big)\,x^8+\big(4\,C_1\,C_2^3+12\,C_3\,C_1^2\,C_2+4\,C_4\,C_1^3\big)\,x^7+\big(6\,C_2^2\,C_1^2+4\,C_3\,C_1^3\big)\,x^6+4\,C_1^3\,x^5\,C_2+C_1^4\,x^4$$

$$FP_5 := \big(5\,C_1^4\,C_4+20\,C_1^3\,C_3\,C_2+10\,C_1^2\,C_2^3\big)\,x^8+\big(5\,C_1^4\,C_3+10\,C_1^3\,C_2^2\big)\,x^7+5\,C_1^4\,x^6\,C_2+C_1^5\,x^5$$

$$FP_6 := 3\,C_1^4\,\big(5\,C_2^2+2\,C_1\,C_3\big)\,x^8+6\,C_1^5\,x^7\,C_2+C_1^6\,x^6$$

$$FP_7 := 7\,C_1^6\,x^8\,C_2+C_1^7\,x^7$$

$$FP_8 := C_1^8\,x^8 \tag{3}$$

For $F(x)$ to be $\exp_H\big(\log_A(x)\big)$, the following series must vanish.

```
> U := collect(expand(add(LH(n)*FP[2^n] - LA(n)*x^(2^n),n=0..N)),
  x);
```

$$U := \Big(6\,v_2\,C_4\,C_1^2\,C_2+6\,v_2\,C_1\,C_3\,C_2^2+3\,v_1^3\,C_1\,C_3\,C_2^2+\frac{1}{254}\,v_3+\frac{1}{2}\,v_1\,C_4^2+\frac{1}{2}\,v_2\,C_2^4+C_8+\frac{1}{8}\,C_1^8\,v_1^7-\frac{1}{508}\,v_1\,v_2^2-\frac{1}{3556}\,v_1^4\,v_2+\frac{1}{2}\,C_1^8\,v_3+3\,v_1^3\,C_4\,C_1^2\,C_2+2\,v_2\,C_1^3\,C_5+3\,v_2\,C_1^2\,C_3^2+\frac{1}{7112}\,v_1^7+v_1\,C_3\,C_5+v_1\,C_2\,C_6+v_1\,C_1\,C_7+\frac{1}{4}\,v_1^3\,C_2^4+v_1^3\,C_1^3\,C_5+\frac{3}{2}\,v_1^3\,C_1^2\,C_3^2+\frac{1}{4}\,C_1^8\,v_1\,v_2^2+\frac{1}{4}\,C_1^8\,v_1^4\,v_2\Big)\,x^8+\Big(v_1\,C_1\,C_6+v_1\,C_3\,C_4+2\,v_2\,C_4\,C_1^3+C_7+v_1^3\,C_4\,C_1^3+v_1\,C_2\,C_5+6\,v_2\,C_3\,C_1^2\,C_2+v_1^3\,C_1\,C_2^3+2\,v_2\,C_1\,C_2^3+3\,v_1^3\,C_3\,C_1^2\,C_2\Big)\,x^7+\Big(C_6+2\,v_2\,C_3\,C_1^3+v_1\,C_1\,C_5+v_1\,C_2\,C_4+v_1^3\,C_3\,C_1^3+\frac{3}{2}\,v_1^3\,C_2^2\,C_1^2+\frac{1}{2}\,v_1\,C_3^2+3\,v_2\,C_2^2\,C_1^2\Big)\,x^6+\Big(C_5+v_1\,C_1\,C_4+v_1^3\,C_1^3\,C_2+2\,v_2\,C_1^3\,C_2+v_1\,C_2\,C_3\Big)\,x^5+\Big(C_4+v_1\,C_1\,C_3+\frac{1}{2}\,v_1\,C_2^2+\frac{1}{2}\,C_1^4\,v_2+\frac{1}{4}\,C_1^4\,v_1^3+\frac{1}{14}\,v_2-\frac{1}{28}\,v_1^3\Big)\,x^4+\big(C_3+v_1\,C_1\,C_2\big)\,x^3+\Big(C_2+\frac{1}{2}\,v_1\,C_1^2+\frac{1}{2}\,v_1\Big)\,x^2+\big(C_1-1\big)\,x \tag{4}$$

This gives the following values for the coefficients:

```
> sol := solve(identity(U=0,x),{seq(C[i],i=1..p^N)});
```

$$sol := \Big\{C_1=1,\ C_2=-v_1,\ C_3=v_1^2,\ C_4=-\frac{12}{7}\,v_1^3-\frac{4}{7}\,v_2,\ C_5=\frac{18}{7}\,v_2\,v_1+\frac{26}{7}\,v_1^4,\ C_6=-\frac{59}{7}\,v_1^5-\frac{57}{7}\,v_2\,v_1^2,\ C_7=\frac{137}{7}\,v_1^6+\frac{163}{7}\,v_1^3\,v_2+\frac{8}{7}\,v_2^2,\ C_8=-\frac{64}{127}\,v_3-\frac{291958}{6223}\,v_1^7-\frac{830513}{12446}\,v_1^4\,v_2-\frac{126023}{12446}\,v_1\,v_2^2\Big\} \tag{5}$$