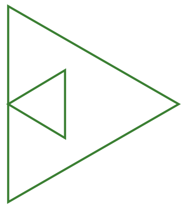
The University Of Sheffield.

SCHOOL OF MATHEMATICS AND STATISTICS **Spring Semester 2019–2020**

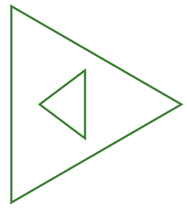
Algebraic Topology **One hour thirty minutes**

This is an open book exam.
Answer ***both*** *questions.*
The submission deadline is 10 am (BST), twenty-four hours after the exam is released. Late submission will not be considered without extenuating circumstances. It is expected that you will be able to complete this exam in approximately one and a half hours and it is recommended that you submit the work within four and a half hours. You will not be penalised for taking longer, however.
Unless it is explicitly stated otherwise, it is intended that calculations are performed by hand (possibly with the aid of a calculator). To gain full marks, you will need to show your working. You will not get full marks if you simply write down output from a computer package.
By uploading your solutions you declare that your submission consists entirely of your own work, that any use of sources or tools other than material provided for this module is cited and acknowledged and that no unfair means have been used.

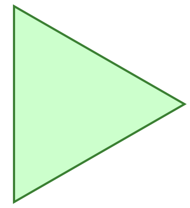
1 Consider the following spaces:



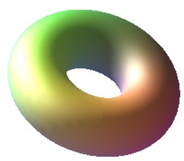
X_0



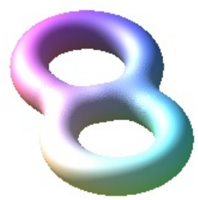
X_1



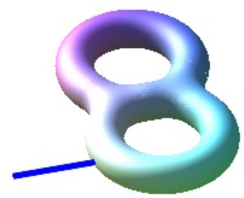
X_2



X_3



X_4



X_5

$X_6 = (S^1 \times S^1) \setminus \{(1,1)\}$ $\qquad$ $X_7 = GL_2(\mathbb{R}) = \{A \in M_2(\mathbb{R}) \mid \det(A) \neq 0\}$

$X_8 = \mathbb{R}$ $\qquad$ $X_9 = \{(u,v) \in \mathbb{C}^2 \mid 1 \leq |u| \leq 2 \leq |v| \leq 3\}$.

(Here X_3 and X_4 are closed orientable surfaces, and X_5 is the union of X_4 with a line segment with one endpoint lying on X_4. Everything else should be clear.)

(a) These 10 spaces can be grouped into 5 pairs $\{X_i, X_j\}$ such that X_i is homotopy equivalent to X_j. Find these pairs, and justify your answers. In each case you should prove that X_i is homotopy equivalent to X_j, and also that it is not homotopy equivalent to any of the other spaces. ***(25 marks)***

(b) For each pair $\{X_i, X_j\}$ as in (a), prove that X_i is not homeomorphic to X_j. (In one case you may need to appeal to some geometric intuition, but you should be able to give a more formal proof in the other four cases.) ***(15 marks)***

2

(a) Let A and B be finite abelian groups such that $|A|$ and $|B|$ are coprime.

(i) What can you say about homomorphisms from A to B? ***(10 marks)***

(ii) Now suppose we have a short exact sequence $A \to U \to B$ of abelian groups. By considering the classification of finite abelian groups, or otherwise, what can you say about U? ***(15 marks)***

(b) Let X be a topological space, with open subspaces U and V such that $X = U \cup V$. Suppose that U, V, X and $U \cap V$ are all path-connected, and that for all $k > 0$ we have $H_k(U \cap V) = \mathbb{Z}/2^k$ and $H_k(U) = \mathbb{Z}/3^k$ and $H_k(V) = \mathbb{Z}/5^k$. Calculate $H_*(X)$. ***(15 marks)***

End of Question Paper