EXAMINABLE CONTENT FOR MAS61015 (ALGEBRAIC TOPOLOGY)

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This is the short version of this document, listing only the points from the notes that are starred (as explained below). There is also a long version that lists all definitions and results in the notes, whether or not they are starred.

For starred definitions, you may be required to state the definition in the exam. For singly starred results, you may be required to state the result in the exam. For doubly starred results, you may be asked to reproduce the proof in the exam. You may also be asked to prove something where the required argument is a step in one of these specified proofs.

Although you will not be examined directly on the unstarred points, understanding those points may nonetheless be helpful for solving problems posed in the exam.

- * Simplex (Definition 1.3)
- * Topology, topological space (Definition 3.13)
- * The metric topology (Example 3.14)
- * The discrete and indiscrete topologies (Example 3.15)
- * Continuous map (Definition 3.19)
- ** Composites of continuous maps are continuous (Proposition 3.24)
- * Subspace topology (Definition 3.25)
- ** The subspace topology is a topology (Proposition 3.26)
- ** Maps into subspaces with the subspace topology (Proposition 3.29)
- ** Closed patching (Proposition 3.35)
- * Homeomorphism, homeomorphic (Definition 4.1)
- ** An open ball is homeomorphic to euclidean space (Example 4.8)
- ** $\mathbb{R}^n \setminus \{0\}$ is homeomorphic to $(0,\infty) \times S^{n-1}$ (Example 4.9)
- ** A continuous bijection that is not a homeomorphism (Example 4.10)
- * Join, reversal and constant paths (Definition 5.6)
- ** Validating path operations (Remark 5.7)
- * Relation of points being connected (Definition 5.8)
- ** The relation based on paths is an equivalence relation (Proposition 5.9)
- * The set $\pi_0(X)$ (Definition 5.10)
- ** Convex sets are path connected (Example 5.11)
- ** Spheres are path connected (Proposition 5.14)
- ** Disconnection results from the Intermediate Value Theorem (Proposition 5.15)
- * Functor (Definition 6.7)
- * Inverse, isomorphism (Definition 6.12)
- ** Inverses are unique (Lemma 6.13)
- ** Functors preserve isomorphisms (Proposition 6.17)
- ** Maps from a coproduct to another space (Proposition 7.3)
- * Product topology, boxes (Definition 7.6)
- ** The product topology is a topology (Proposition 7.9)
- ** Maps to a product from another space (Proposition 7.11)
- * Quotient topology (Definition 7.17)
- ** The quotient topology is a topology (Proposition 7.18)
- ** Maps from a quotient space to another space (Proposition 7.19)
- ** π_0 preserves products (Proposition 7.30)
- * Hausdorff separation, Hausdorff space (Definition 8.1)
- ** Metric spaces are Hausdorff (Example 8.2)

** Subspaces of Hausdorff spaces are Hausdorff (Proposition 8.6)
** Products of Hausdorff spaces are Hausdorff (Proposition 8.8)
* Compact space, open cover, finite subcover (Definition 8.9)
** Closed subspaces of compact spaces are compact (Proposition 8.17)
* Lebesgue number (Definition 8.30)
* Homotopy (Definition 9.2)
** Being homotopic is an equivalence relation (Proposition 9.3)
* The category hTop (Definition 9.9)
* Homotopy equivalence (Definition 9.10)
** Convex sets are contractible (Example 9.14)
* Homotopy retract, homotopy retraction (Definition 9.22)
** Homotopy retracts of contractible spaces are contractible (Proposition 9.24)
** π_0 is homotopy invariant (Proposition 9.25)
* The set $S_k(X)$ of singular k-simplices (Definition 10.1)
* Linear simplices (Example 10.4)
* The group $C_k(X)$ of singular k-chains (Definition 10.8)
* The maps $\delta_i\colon \Delta_{n-1} \to \Delta_n$ (Definition 10.12)
* The algebraic boundary map $\partial\colon C_n(X) \to C_{n-1}(X)$ (Definition 10.14)
** $\partial^2 = 0\colon C_4(X) \to C_2(X)$ (Example 10.20)
* The homology group $H_k(X)$ (Definition 10.21)
** Homology of a point (Proposition 10.23)
* Homomorphisms between cyclic groups (Lemma 12.1)
* The Chinese Remainder Theorem (Proposition 12.2)
* Primary decomposition of cyclic groups (Corollary 12.3)
* Finitely generated abelian group (Definition 12.4)
** $\mathbb{Q}$ is not finitely generated (Example 12.7)
* Exact sequence (Definition 12.12)
** Various properties of exact sequences (Proposition 12.15)
** Orders in short exact sequences (Lemma 12.20)
* Chain complex, cycles, boundaries, homology (Definition 13.1)
* Chain map between chain complexes (Definition 13.9)
** Z_*, B_* and H_* as functors of chain complexes (Proposition 13.11)
* Homology as a functor on spaces (Construction 13.12)
** Homological effect of constant maps (Remark 13.14)
* Chain homotopy (Definition 14.1)
** Chain homotopic chain maps have the same effect in homology (Proposition 14.7)
** Homotopy equivalences give isomorphisms in homology (Corollary 14.14)
* The Mayer-Vietoris sequence (Theorem 15.1)
** Homology of spheres (Theorem 15.3)
** Different spheres are not homotopy equivalent (Theorem 16.1)
** Different euclidean spaces are not homeomorphic (Theorem 16.2)
** The Brouwer theorem: any $f\colon B^n \to B^n$ has a fixed point (Theorem 16.3)
* Snakes (Definition 17.3)
* The connecting map δ for the Snake Lemma (Definition 17.6)
** Mayer-Vietoris exactness III (Proposition 17.11)
** Homology of convex open set with removed points (Proposition 20.2)
** Homology of the torus (Proposition 20.5)
** Homology of the projective plane (Projective 20.7)
* The Generalised Jordan Curve Theorem (Theorem 21.1)
* Covering map, trivially covered set (Definition 22.2)
* The transfer map (Definition 23.6)
** The transfer is a chain map (Proposition 23.8)
* The Borsuk-Ulam theorem: no antipodal maps $S^{>m} \to S^m$ (Theorem 24.5)