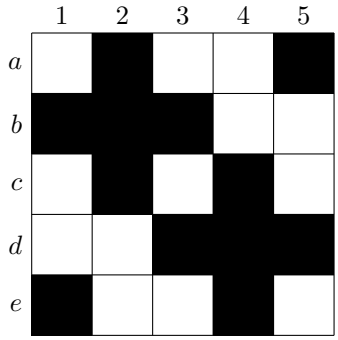
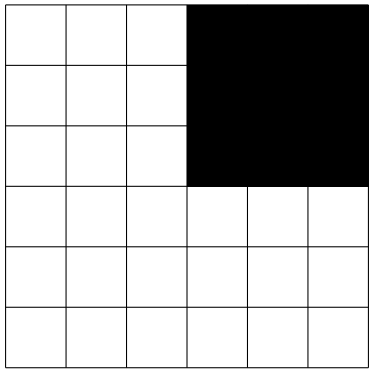
MAS334 COMBINATORICS — PROBLEM SHEET 4

Please hand in exercises 4.1 and 4.4 by the end of Week 9.

Exercise 4.1. Use the tabular method to find all full matchings for the following board:



Exercise 4.2. Use Remark 10.7 to find the number of full matchings for the following board B:



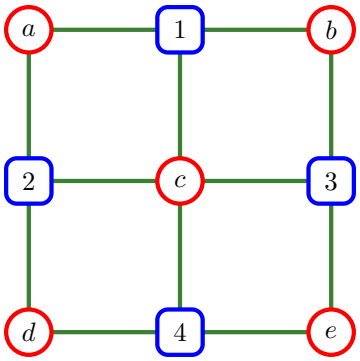
Can you explain the answer in a different way?

Exercise 4.3. Let B be an $n \times n$ board (with $n > 0$), and let $\overline{B}$ be the complement. Theorem 10.3 and Remark 10.7 tell us that

$$c_n(B) = \sum_{k=0}(-1)^k(n-k)!c_k(\overline{B})$$
$$c_n(\overline{B}) = \sum_{k=0}(-1)^k(n-k)!c_k(B).$$

Check these equations directly in the case where B is the full board F_n.

Exercise 4.4. Consider the matching problem where $A = \{1, 2, 3, 4\}$ and $B = \{a, b, c, d, e\}$ and the incidence graph is as follows:



(a) Find a very plausible subset $U \subseteq B$.
(b) Find a barely plausible subset $V \subseteq B$ with $V \neq \emptyset$.
(c) Does the matching problem have a solution?

Exercise 4.5. Consider a matching problem where $B = \{1, 2, \ldots, n\}$ and $|C_k| \geq k$ for all k. Prove that the problem is solvable.

Exercise 4.6. Consider a job allocation problem where $|A| = |B| = n$, and every subset $U \subseteq B$ is barely plausible. What can we conclude?