Uncertainty in Probabilistic Risk Assessment: A Review

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1 Introduction

The concept of risk is invoked as a central issue across a wide range of policy debates. How can risks be reliably identified, how can they be managed, under what circumstances should they be accepted or rejected and, especially, how are they likely to be interpreted or 'perceived' by different people? These questions arise in areas as diverse as health and lifestyle, hazardous industries, pensions and investments, transport, climate change and environmental protection.

Risk assessment is widely recognised as a systematic process for quantitatively (or qualitatively) describing risk. Risk is commonly described as a combination of the *likelihood* of an undesirable event (accident) occurring and its consequences. Alternatively, Kaplan (1997) expressed risk as a mathematical combination of an accident's event probability of occurrence and the consequence of that event should it occur.

Bedford and Cook (2001) characterise risk with two particular elements: hazard (a source of danger) and uncertainty (quantified by probability). Probabilistic risk assessment (hereafter shown by PRA) specifically deals with events represented by low probabilities of occurring with a high level of consequence. The elicitation of probability distribution from expert(s) is one of the key research areas in PRA which will be presented in this review. The distinction between sources of uncertainty often comes into play in the elicitation of these probabilities. Uncertainties are sometimes distinguished as being either *aleatory* or epistemic. We study this distinction, and present the definitions of these types of uncertainty in Section 2.

In Section 3, we present a review of the literature on the use of expert opinion (with emphasis on single expert) in PRA.

Some applications of probability elicitation in PRA will be introduced in Section 4.

2 Aleatory and Epistemic Uncertainties in Probabilistic Risk Assessment

In this section, we shall study different types of uncertainty mentioned above, and how they could be reduced in probabilistic risk analysis. There are different perspective about the distinctions between the types of uncertainty which will be presented in this section.

Definition 2.1 Aleatory Uncertainty: This uncertainty arises because of natural, unpredictable variation in the performance of the system under study.

The knowledge of experts cannot be expected to reduce aleatory uncertainty although their knowledge may be useful in quantifying the uncertainty. Thus, this type of uncertainty is sometimes referred to as irreducible uncertainty.
Several Psychologists including Kahneman and Tversky (1982) and Pitz and Wallsten (2000) believed that aleatory (or external) uncertainty reflects expert's direct experience with events that vary (*more comments should come here*).

Definition 2.2 Epistemic Uncertainty: This type of uncertainty is due to a lack of knowledge about the behaviour of the system that is conceptually resolvable.

The epistemic uncertainty can, in principle, be eliminated with sufficient study and, therefore, expert judgments may be useful in its reduction. From psychology point of view, epistemic (or internal) uncertainty reflects the possibility of errors in our general knowledge. For example, one believes that the population of city A is less than the population of the city B, but one is not sure of that.

The distinction between these types of uncertainty is useful because, as it is mentioned above, epistemic uncertainty is reducible. Oakley and O'Hagan (2002) suggested that sensitivity analysis can help us to explore which part of the uncertainty in the model output is

removable. They also argued that parameter uncertainty is generally epistemic, because we simply do not know what are the correct values for the input parameters. However, sometimes there is an aleatory component where we want to forecast the real-world process by a model in which some of the conditions specified in the inputs are not controlled and specified (see Oakley and O'Hagan (2002) for further details). However, Winkler (1996) claimed that they basically refer to the same distinctions, but their distinctions have useful implications for the practice of modelling and analysing complex system.
Thus, we can conclude that there is increasing interest in distinguishing types of uncertainty in PRA.
Apostolakis (1988) gave boost to Winkler's claim by the following quotation: "probability is fundamentally the same concept regardless of whether it appears in the model of the world or in the subjective distributions for the parameters. There is only one kind of uncertainty stemming from our lack of knowledge concerning the truth of a proposition, regardless of whether this proposition involves the possible values of the hydraulic conductivity or the number of earthquakes in a period of time. Distinctions between probabilities are merely for our convenience in investigating complex phenomena. Probability is always a measure of degree of belief."

Indeed, Apostolakis (1988), Winkler (1996), and Oakley and O'Hagan (2002) stresses this point that probability is the only way to represent uncertainty regardless of practical difficulties in doing it accurately.

Hofer (1996) by a simple example illustrate when to separate and when not to separate uncertainties mentioned above. He concluded that separation of uncertainties is costly because

- there is effort involved in consistent separation.
- expert judgment elicitation has to account for separation.
- propagation through the model needs to happen separately.
- sensitivity measures are of interest in both dimensions.

Hofer et al (2002) approximated epistemic uncertainty in present of aleatory uncertainty by two simple Monte Carlo simulations which requires much less computational effort.
They represented any scalar process variable or model outcome Y in terms of joint epistemic and aleatory uncertainties as follows

$$Y = h(U, V)$$

where $U = \{\text{all epistemic uncertainties (uncertain parameters)}\}$,
$V = \{\text{aleatory uncertainties (stochastic variables)}\}$, h is the computational model considered as a deterministic function of both uncertainties mentioned above.
When holding the epistemic variable U fixed at a value u, that is, $U = u$, the resulting output Y is uniquely a function of V. Therefore the conditional expected value of Y given $U = u$, i.e. $E(Y \mid U = u)$ can be considered as a representative scalar value to quantify the aleatory uncertainty in Y. Therefore, the principle aim of an epistemic uncertainty analysis from models subject to both epistemic and aleatory uncertainties is to determine the subjective probability distribution of $E(Y \mid U)$ for the possible values of u. This can be done by approximation of the first two central moments of the distribution of $E(Y \mid U)$, i.e. $E(E(Y \mid U))$ and $var(E(Y \mid U))$. They used two-stage nested Monte carlo simulation to approximate these moments (for further details see Hofer et al (2002)).

Hora (1996) believed that the distinction between the sources of uncertainty depends on the objective of the study. He reported that the distinction between the types of uncertainty cannot be purely made through physical properties or the expert judgments. The same quantity in one study may be treated as having aleatory uncertainty while in another study the uncertainty maybe treated as epistemic.

To design a probability elicitation process, experts should acknowledge the need to separate types of uncertainty. Questions should be constructed so that these uncertainties retain distinct representations. Often, this is accomplished by using conditional probability distributions for the quantities having aleatory uncertainty and marginal probability distributions for quantities having epistemic uncertainty. The same strategy can be used for quantities embodying both types of uncertainty. To ensure that the elicitation process recovers the described information, the questions asked need to be carefully posed. In particular, questions about quantities that are integrated or averaged over time or space should be carefully constructed to decompose the elicitation quantities so that the two types of uncertainty are appropriately represented (For examples and further details, see Hora (1992, 1996), Bonano, et al (1995)) .

Zio and Apostolakis (1996) studied the influence of these uncertainties in a physical model predictions, and when and how these uncertainties could be removed. They represented the *adjustment factor approach* to quantify the error in (physical) model predictions, which can be related to limitations in the analyst's knowledge. The principle of this approach is to employ the best model available, denoted y^*, and compensate for the error associated with it by introducing a factor e^*. This adjustment factor might be additive (e_a^*) or multiplicative (e_m^*), resulting in

$$y = y^* + e_a^*$$

or

$$y = y^* + e_m^*$$

The factor e^* is generally unknown and the uncertainty associated with it can be represented in the form of a distribution $g(e^*)$. It is important to note that the uncertainty in e^* could be of the epistemic type only or both epistemic and aleatory. In the first case, e^* simply represents the systematic bias of the model prediction and the uncertainty in its numerical value is strictly related to the lack of knowledge which could, in principle, be eliminated with a single observation on y. In the second case, the model bias e^* itself exhibits aleatory variability, due to some random effects which have been neglected in the model. In this case, a single observation cannot eliminate the uncertainty and actually a sequence of observations, repeated under apparently identical conditions, would lead to different values. e^* is then described by an aleatory distribution whose parameters are uncertain due to the epistemic uncertainty in their values.

Kundsen and Smith (2000) have addressed the incorporation of uncertainty specific to the logic models themselves. They applied their approach to investigate the uncertainty in the unreliability of a nuclear power plant auxiliary feedwater (AFW) system. They constructed AFW fault tree to evaluate both aleatory and epistemic uncertainty. The following general results can be picked up from this research.

They showed that the aleatory uncertainty can be separated into two basic parts: one part represents the underlying probability model while the other part represents the *applicability* of the underlying probability model.
The underlying probability model usually requires parameters such as a failure rate of a component in AFW system. For these parameters, epistemic uncertainty based upon the collected data from the laboratory was assigned. Then, they presented results associated with uncertainty analysis consists of measurements for epistemic (parameter) uncertainty, aleatory uncertainty and combined measure of them in terms of their means, 5^{th}, 50^{th} and 95^{th} percentiles.

To conclude studies about treatment of aleatory and epistemic uncertainty in complex engineered system, the following remarks can be obtained from engineering risk assessments papers (some of them mentioned above):

- Reducing uncertainties (related to ignorance and sample size) and aleatory uncertainties should be clearly distinguished.
- Quantitative risk estimates, if presented, should be expressed in terms of distributions rather than as point estimates.
- Through use of probabilistic risk assessment techniques, important uncertainties have been and continue to be brought into better focus and may even be reduced compared to those that would remain with sole reliance on deterministic decision-making.
- Probabilistic risk assessment for complex engineered facilities involve aleatory uncertainty due to the many different types of accidents that can occur and epistemic uncertainty due to the inability of the analysts involved to precisely determine the frequency and consequences of these accidents.

2.1 Uncertainty and Variability

It is very important in PRA to know the differences between *uncertainty* and *variability.* We first briefly introduce both of them, and then discuss some of their differences.

Ucertainty, as explained above, represents partial ignorance or the lack of perfect knowledge on the part of the analyst. Uncertainty[1] may be reduced by further measurement. Variability represents heterogeneity in a population that is irreducible by additional measurements. Variability is an objective property of a population. In fact, a clear definition requires an unambiguous description of the population.

Two primary areas where we need to distinguish between these two concepts are when we must:

- explain model results to decision makers and the public.
- expend resources for data collection.

It should be noticed that there is no compelling reason to distinguish between variability in observations and uncertainty due to the lack of knowledge when the assessment endpoint is a fixed quantity. But when the assessment endpoint is a stochastic variable representing a distribution of true values in a population, then uncertainty due to lack of knowledge about this distribution must be distinguished separately from stochastic variability. In the first case, uncertainty is represented by a subjective probability distribution composed by alternative realization of a true but unknown fixed quantity. From this distribution a best estimate and subjective confidence interval can be obtained. In the second case, uncertainty is represented by alternative realisation of the true (but unknown) distribution. From these alternative distributions, a best estimate and a subjective confidence interval can be achieved for each fractile and for the mean value (for further details, see Anderson and Hattis (1999)).

3 Expert Judgments Process in Probabilistic Risk Assessment

Generally, in risk analyses with little or no relevant historical data[2], expert judgment is required. How to use such judgment depends on which probabilistic method should be used to assess risk. For example, Aven (2000) and Apeland and Aven (2000) used a purely classical method and a combined classical (frequentist) and Bayesian approach, respectively. Under these approaches, establishing estimates of true statistical quantities (parameters), such as,

[1] More precisely, epistemic uncertainty

[2] It is known as *hard data* in the most risk assessment papers.

probabilities and failure rates would be main focus in the risk analysis.
If sufficient data are available, the estimation of parameters can be obtained based on analysis of the classical statistics approach only. But, if we have scarce data[3] the combined classical and Bayesian approach will be suggested which allows us to use of expert judgment to establish subjective[4] uncertainty measures associated with the true values of the parameters of interest.
It should be noticed that in both approaches, there are uncertainties associated two levels: the occurrence of future events; and the true values of the probabilities and failure rates.
One can find the methods explained to deal with risk assessment are not very appropriate (Aven (2000) and Apeland and Aven (2000) for further details). In fact, the use of expert opinion is very a well-known way to obtain these levels.

In this section, we review the general methods of eliciting subjective probabilities from expert(s) to use in probabilistic risk assessment. We present a review of the methods to incorporate expert judgment in PRA. This includes the use of the expert judgment elicited from several experts. We also briefly point out some cognitive factor that are important in the elicitation of expert opinion.

3.1 Introduction to Expert Judgment and how to Select Experts

Expert opinions can provide useful information for forecasting, making decision, and assessing risks. Expert judgment can be considered as an informed assessment or estimate, bases on the experts's training and experience, about an uncertain quantity or quality of interest. Expert judgments are required in most steps of risk assessments: hazard identification, risk estimation, risk evaluation and analysis of options.

[3]In quantifying risks, the prior position is often a scarce amount of necessary information (or data) on component failure or specific event.

[4]For instance, Apeland et al (2002) developed an approach based on subjective probability only, called *predictive epistemic*[5]. In this approach, subjective probability is considered as a measure of uncertainty related to predictions of observable quantities [6]. Risk assessments are carried out as a support for decision making on major hazards activities. The risk assessments are explanatory in nature and are not aiming at predictions of accidents which might occur. In different cases enormous experience has been obtained with risk evaluations by using well-known approaches, such as, fault trees.

The use and elicitation of expert judgment is therefore subject to on-going research. So, it is important to clarify the meaning of the words expert and judgment which have been used in the risk communities. An expert is a person with special knowledge or skills in a particular domain. Some evidence that might be used as criteria for selecting expert(s) are: experience in performing judgements and making decisions, based on evidence of expertise, e.g. degrees, research, publications, positions and experience, awards, etc; availability and willingness to participate; impartiality and inherent qualities like self-confidence and adaptability.

Judgment refers to inferences made in forming opinions. Thus, an expert judgment should be the inferential opinion of a domain specialist regarding an issue within the area of expertise. The judgment is obtained through a formal elicitation process (discuss below) that seeks to minimise biases (availability, anchoring and adjustment, representation, motivational biases [7] , etc. (see Daneshkhah (2004) and references therein)) and to help the expert construct the subjective probability distribution.

The use of expert judgment is subject to a developing set of rules which include:

- Experts are capable of expressing useful opinions as probability distributions. Usually these are what we would call uncertainty rather than variability, although sometimes both are confused.
- Effort must be made to reduce or account for the biases of experts. Overconfidence seems to be the most important of these for risk analysis.
- Expert opinions are almost surely dependent[8], in contrast to the mistaken notion of identical, independently distributed observations. It is arguable that experts should be dependent to some extent.
- It should be noticed we want experts to look at the problem from very different point of views.

[7]Mosleh et al (1988) reported that while there is evidence that expert opinion can be highly beneficial in probabilistic risk analysis, little attention has been paid to structuring the elicitation process.

[8]Some modelling is needed to account for dependence, especially if opinions from multiple experts on the same event or variable are to be aggregated.

In the probabilistic risk assessment, expert opinion is used in two ways:

- To structure a problem. Experts determine which data and variables are relevant for analysis, which analytical methods are appropriate and which assumptions are valid.
- To provide estimates. For example, experts may estimate failure or incidence rates, determine weighting for combining data sources, or characterize uncertainty.

Some general principles of expert elicitation that are recommended in risk assessment literature are:
1) Only expert's opinion is worth eliciting.
2) Experts are more comfortable and able to answer questions about observable.
3) Elicitation should involve feedback, to better estimate the uncertainty in the elicited distribution.
4) Elicitation should involve asking both unconditional and conditional questions on hypothetical observed data.
5) If the expert opinion systematically represents overconfidence due to anchoring or measurement error, consider this possibility into the elicitation method.
6) If there is doubt about the ability of the experts to estimate their own uncertainties, this issue should be considered into the elicitation method.

3.2 Expert Judgement on risk model structure

The main differences in risk assessment approaches relate to how uncertainties pertaining to risk model parameter values, model structure and completeness are addressed. The uncertainties are dependent on the complexity and understanding of the causal and/or logical relationships of quantities and/or events of the real world (see Nielsen and Aven (2003) for further details and examples), and their significance in the decision-making process depends on the adopted decision rules and criteria.

A risk model is basically a model of uncertainties related to a real system. How these uncertainties are addressed qualitatively and quantitatively affects the completeness and the credibility of the risk assessment. The Bayesian statistical approach (Gelman et al (1995), Lindley

(2000)), the alternate hypthesis approach (Zio and Apostolakis (1996)), the adjustment factor approach (Zio and Apostolakis (1996), discussed above), and the predictive Bayesian approach (Nilsen and Aven (2003)) addressed uncertainties in their own ways. This should be taken into consideration in the definition of the decision criteria adopted in the decision context, suggested implicitly by authors listed above.

3.3 Use Expert Opinion in Practice

As we mentioned above, risk analysts have traditionally used historical data as information basis for frequently and probability assessments. A large number of databases are established for this purpose and a major subtask of piratical analysis is collecting the data from these and from some other sources. An important factor which is often causing relevant data to be scarce is that risk analyses typically deals with rare events. Furthermore, the systems under study often represents new concepts and arrangements which little or no experience exists, and use of expert judgments is enforced. In other words, we could say the expert judgment is typically appropriate when:

- Data are sparse or difficult to obtain. Sometimes information is not available from historical records, prediction methods or literature.
- Data are too costly to obtain.
- Data are open to different interpretations, and the results are uncertain (unstable). Models to analyse risks are not available.
- There is a need to perform an initial screening of problems.

However, there are some objections among some risk analysts associated with using the subjective probability, provided by expert, as a measure of uncertainty in risk analysis. They believe that these judgments are superficial and imprecision. Experts' ability to express their uncertainty in terms of probabilities and whether such statements constitute a sufficiently credible basis in the elicitation and decision-making contexts are discussed extensively in the literature. A large portion of this work is basis on so-called heuristics and biases (see Daneshkhah (2004) and references therein).

Risk analysts may practically express these biases as mechanisms that:

- lead to inconsistency between the expert's system knowledge and his/her assessment of uncertainty, or

- introduce a disparity between the perceived uncertainty and the probability figure which is eventually obtained from expert.

Skjong and Wentworth (2001) reported that experts are subject to similar biases as lay people. Even though they may not be influenced in the same manner or the same extent. They concluded that expert judgments should be utilised with great caution.
Sjoberg (1980) presented a work regarding to misunderstanding that might be appeared in communications of risk between lay people. Since, the definition of risk utilised in risk assessments is not a universal definition. The word *risk* is known to be ambiguous and many more or less specific definitions have been attributed to it. He claimed that there are three broad classes of meaning involved in PRA:

- Probability of (negative) complement events

- Concern of the consequence of the complement events themselves

- Joint probability distribution and consequence of events

The product of probability and consequence is referred as the definition of risk associated with an event in typical risk assessments, it may not be surprising that numerous misunderstandings appear in communications of risk, as perceived risk may not be well represented by this product.

3.4 Aggregating judgments of Multiple Experts

In selection of experts, a decision must be made whether to have a single expert or a group of experts. According to Clemen and Winkler (1999) combination or aggregation approaches may be cataegorised as mathematical and behavioural approaches. Mathematical approaches consists of processes (models) that combine or adjust the individual probability values or distributions into one single value or distribution. These approaches range from summary measures, e.g., arithmetic or geometric means, to procedures requiring inputs with respect to quality and dependence of the experts' probabilities.

Behavioural approaches attempt to generate agreement among the experts by various ways of interactions. The interaction may involve face to face interactions or unidentifiable exchange of information. These approaches consider the quality and dependence of the experts probabilities implicitly.
The mathematical and behavioural methods have found to be similar in performance, with the mathematical rules having a slight edge (see Clemen and Winkler (1999) and Ouchi (2004)). Mosleh et al (1988) also reported that although empirical evidence indicates that mathematical methods of aggregation generally yield better results than behavioural methods, the latter methods are often perceived appealing, particularly when experts have knowledge in different areas and the synthesis of their expertise is needed.

Because experts should be dependent, care must be taken in aggregation their opinions. Aggregating is preferable to carrying through the opinion of each expert and letting the decision maker decide how to aggregate the posteriors. However, aggregating should be based on rational considerations and may involve making assumptions about the independence or dependence of parts of the model (Kaplan and Burmaster (1999)).

Rosqvist (2000) to aggregate experts' judgments for specifying the failure intensity function of a repairable system, developed a hierarchical Bayesian expert model. Expert judgment is elicited in the form of numbers of failures in the time windows chosen by the experts, and then aggregated by the Bayesian model mentioned above.

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